Differential Equations

N T0 VERSION Y0 a b epsilon b Crop PrettyPrint Beder Set Constraints Standardige Domain Cosine Transform Summation f Express BidODE ForPiot Prepare DEQ Test Run DiffEg Solution Test Save Expression As JSON Load Expression From JSON	S0 0 2019 0221 0 9 6 0.196 2.7 1628 1828459 0.05	Select an equation "Run DiffEQ Solution Test" From the Function drop-down menu
Solution Test Function Equation Description Approximation a b x y OK Cancel	TeldODE y'(x)+b*y(x)-a=0 Field Equation FODE 8.8 0.196 Domain Parameter Reload Spline RK4	The "Solution Test" tool is shown The information about the equation is shown The required parameters are shown with their values The RK4 button starts the Runge-Kutta tool
Solution Approximation Function Equation Description Approximation Y0 h T0 N OK Cancel	FieldODE X y'(x) + b * y (x) - a = 0 Field Equation FODE 10 0.05 0 30 Back Next	The "Solution Approximation" screen shows the set values of the RK4 formula parameters Press the OK button to run the T0 approximation



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Test the Solution Quality

Solution Test Function Equation Description Approximation a b x y OK Cancel	fieldODE y'(x) + b'y(x) - a = 0 Field Equation FODE 9.8 0.196 Domain Parameter Function Spline Reload Spline	The Solution Test screen now shows the solution "y" has a "Function Spline" available for test
Function Plot Parameters t ○ ○ ← ▼ 2 ↔ 0.01 J OK Cancel		The OK button show a form for entering domain parameters
IncidODE (04, [0 <= x <= 2]	10 -= x -= 2]	A plot of the solution error for the domain is shown An error spike is typically seen at each knot

Test function as Solution

CALCLIB Environment Notation Value Data Scripts Symbols Functions ADSplineTwoDs.td N/PHNITY wo 1000000.0 AngCatClaston.td N/PHNITY wo 1201902.01 AngCatClaston.td N/PHNITY wo 2019.02.01 AngCatClaston.td AngPedication.c 2719281828459 Pedication.c AngPedication.td AngPedication.c 2719281828459 Pedication.c AngPedication.td Function Pedication.c 2719281828459 AngPedication.td Pedication.c Pedication.c 2719281828459 AngPedication.td Pedication.c Pedication.c Pedication.c AngPedication.td Pedication.c Pedication.c Pedication.c Pedication.c AngPedication.td Pedication.td Pedication.c Pedication.c Pedication.c Pedication.c BernoulliObE.td Pedication.td Pedication.com Pedication.com Pedication.c Pedication.c Pedication.com Pedication.com Pedication.com Pedication.com Pedication.com Pedication.com Pedication.com Pedication.com <t< th=""><th>Select function "Prepare DEQ Test" from function menu A dialog will request the formal parameter name for the function within the differential</th></t<>	Select function "Prepare DEQ Test" from function menu A dialog will request the formal parameter name for the function within the differential
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Solution Test		
Function	fieldODE	The solution test form comes up
Equation	y'(x)+b*y(x)-a=0	Now the formal parameter is shown to reference
Description	Field Equation	the selected actual parameter
Approximation	FODE	
a	9.8	Function $y = y10 (x)$
b	0.196	Press OK button
x	Domain Parameter	
Y	y10(x)	
OK. Cancel	Reload Spline RK4	
Function Plot Parameters		Enter domain description
(c) fedd0000 (0), [0 <= x <= 3] Image: Second se		The error plot is displayed



RungeKutta.txt

Use Runge-Kutta approximation and check the error computed against solution vector

// <u>Runge-Kutta</u> error test

// generate interpolation $T = [0 \le t \le N] (T0 + t * h)$ solution = CHEBYSHEV (T, Y)

// generate alias for solution and for derivative
!! y (x) = solution @*^ x
!! y' (x) = solution @*^' x

// improve polynomial use efficiency
OPTIMIZE y; OPTIMIZE y'

// post error test !! testError (x) = y' (x) - f (x, y(x))

// plot error against domain described by test parameters
PLOTF testError [T0 <= x <= T0 + h * N <> h/10]

SIDEBYSIDE "Regression Versus Error Plot"

RungeKuttaRiccati.txt

A simple example of use of **RungeKutta.txt**

// Riccati

!! u(x) = x / 2
Y0 = 0.1; T0 = 0; h = 0.05; N = 30
!! f(Tn,Yn) = Yn^2 + Tn * u(Tn) * Yn + u(Tn)

Y = (0.1, 0.10113171857626335, 0.10354424073431215, 0.10726542307837783, 0.11232678123700574, 0.11876514779425473, 0.12662446126207821, 0.1359577239358084, 0.14682917975976403, 0.15931677951018194, 0.1735150208399999, 0.18953827665792933, 0.20752475923532693, 0.22764131262151488, 0.250089287107206, 0.2751118334633071, 0.303003071635968, 0.3341197537209428, 0.36889627761184174, 0.40786425183380254, 0.45167832094096894, 0.5011507269701952, 0.5572982587771222, 0.6214070869571133, 0.6951239492603989, 0.7805870513163335, 0.8806183876003942, 0.999013878270224, 1.1409946276569765, 1.3139341820639356, 1.5285808552014561)



READ RungeKutta.txt

Equation Renders

RENDER commands are available to supply MathML / LaTeX type renders of sophisticated equations.



Functions found in the Functions list can be rendered using:

RENDERF function-name

As seen above equations can be rendered directly from the command line e.g.

RENDER (1 - x^2) * <u>Tn</u>''(x) - x * <u>Tn</u>'(x) + n^2 * <u>Tn</u>(x) = 0